## SAFE HANDS & IIT-ian's PACE

**MONTHLY MAJOR TEST-09 (JEE) ANS KEY Dt. 05-09-2023**

| PHYSICS | | CHEMISTRY | | MATHS | |
|---|---|---|---|---|---|
| Q. NO. | [ANS] | Q. NO. | [ANS] | Q. NO. | [ANS] |
| 1 | C | 31 | B | 61 | A |
| 2 | A | 32 | B | 62 | B |
| 3 | B | 33 | B | 63 | C |
| 4 | A | 34 | B | 64 | C |
| 5 | C | 35 | C | 65 | D |
| 6 | B | 36 | B | 66 | A |
| 7 | B | 37 | D | 67 | C |
| 8 | B | 38 | A | 68 | C |
| 9 | C | 39 | A | 69 | B |
| 10 | D | 40 | D | 70 | C |
| 11 | C | 41 | C | 71 | C |
| 12 | B | 42 | C | 72 | A |
| 13 | B | 43 | D | 73 | C |
| 14 | A | 44 | C | 74 | C |
| 15 | C | 45 | A | 75 | A |
| 16 | A | 46 | D | 76 | A |
| 17 | C | 47 | D | 77 | C |
| 18 | A | 48 | B | 78 | D |
| 19 | D | 49 | A | 79 | B |
| 20 | C | 50 | B | 80 | C |
| 21 | 2 | 51 | 4 | 81 | 100 |
| 22 | 19.2 | 52 | 6 | 82 | 4 |
| 23 | 5 | 53 | 4 | 83 | 3 |
| 24 | 15 | 54 | 6 | 84 | 4 |
| 25 | 6 | 55 | 6 | 85 | 0 |
| 26 | 35 | 56 | 5 | 86 | 4 |
| 27 | 3 | 57 | 2 | 87 | 3 |
| 28 | 2 | 58 | 2 | 88 | 8 |
| 29 | 0 | 59 | 4 | 89 | 1 |
| 30 | 0.3 | 60 | 1 | 90 | 1 |

# : ANSWER KEY :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) | c | 2) | a | 3) | b | 4) | a |
| 5) | c | 6) | b | 7) | b | 8) | b |
| 9) | c | 10) | d | 11) | c | 12) | b |
| 13) | b | 14) | a | 15) | c | 16) | a |
| 17) | c | 18) | a | 19) | d | 20) | c |

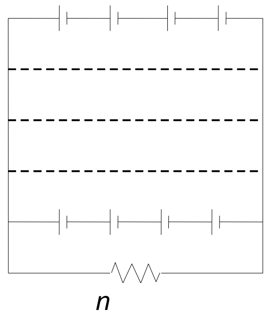
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) | 2 | 2) | 19.2 | 3) | 5 | 4) | 15 |
| 5) | 6 | 6) | 35 | 7) | 3 | 8) | 2 |
| 9) | 0 | 10) | 0.3 | | | | |

# : HINTS AND SOLUTIONS :

1 **(c)**

In mixed grouping the current in the external circuit will be maximum when the internal resistance of the battery is equal to the external resistance,

$R = \frac{mr}{n}$




Given, $R$=3Ω, $r = 0.5\Omega$

$\therefore \quad 3 = \frac{m}{n} \times 0.5$

$\Rightarrow \frac{m}{n} = 6$

$\Rightarrow m = 6n$

Total number of cells= $m \times n = 24$ ...(i)

From Eqs. (i) and (ii), we get ...(ii)

$6n \times n = 24$

$\Rightarrow 6\text{n}^2 = 24$

$\Rightarrow n^2 = 4$

$\Rightarrow n = 2, m = 12$

3 **(b)**

$l \propto \frac{1}{r^2}$, if radius of the wire is doubled then increment in length will become $\frac{1}{4}$ times $i.e. \frac{12}{4} = 3mm$

4 **(a)**

Work done = Area of closed $PV$ diagram

$= (2V - V) \times (2P - P) = PV$

5 **(c)**

$P = Fv = m.\frac{dv}{dt} \cdot v$

$\int v\,dv = \int \frac{p}{mdt}; \frac{v^2}{2} = \frac{pt}{m}$

$v = \sqrt{\frac{2p}{m}} t^{1/2}; \frac{dx}{dt} = \sqrt{\frac{2p}{m}} t^{1/2}$

$\int dx = \sqrt{\frac{2p}{m}} \int t^{1/2} dt;$

$x = \sqrt{\frac{2p}{3}} \frac{t^{3/2}}{3/2} = \frac{2}{3}\sqrt{\frac{2p}{3}} t^{3/2}$

$x \propto t^{3/2}$

6 **(b)**

$n_A = 258\ Hz$

$n_B = 262\ Hz$

Let n is the frequency of unknown tuning fork. It produces x beats with 258 and 2x with 262

262-(258-x)=2x

262-268+x=2x

X=4

N=254Hz

7 **(b)**

$\frac{mg}{m(g-a)} = \frac{3}{2} \Rightarrow a = g/3$

8 **(b)**

For dark fringe at $P$

$S_1P - S_2P = \Delta = (2n-1)\lambda/2$

Here $n = 3$ and $\lambda = 6000$Å

So, $\Delta = \frac{5\lambda}{2} = 5 \times \frac{6000\text{Å}}{2} = 15000\text{Å} = 1.5\ micron$

9 **(c)**

Let student catch the bus after $t$ sec. So it will cover distance $ut$

Similarly distance travelled by the bus will be $\frac{1}{2}at^2$ for the given condition

$ut = 50 + \frac{1}{2}at^2 = 50 + \frac{t^2}{2} \quad [a = 1\ m/s^2]$

$\Rightarrow u = \frac{50}{t} + \frac{t}{2}$

To find the minimum value of $u$

$\frac{du}{dt} = 0$, so we get $t = 10\ sec$, then $u = 10\ m/s$

10 **(d)**

Focal length of lens in given by

$\frac{1}{f_w} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= \left(\frac{1.6}{1.63} - 1\right)\left(\frac{1}{15} + \frac{1}{15}\right)$

$= -\frac{0.03 \times 2}{1.63 \times 15} = \frac{-6}{1.63 \times 15}$

$so, \quad f_w = -\frac{815}{2}$

$= -407.5\ cm$

11 **(c)**

At minimum distance, incidence is normal.

Therefore, $E = \frac{I}{r^2} = \frac{250}{6^2} = 6.94$ lux

12 **(b)**

$W = JQ \Rightarrow \frac{1}{2}\left(\frac{1}{2}mV^2\right) = J \times mS\Delta\theta \Rightarrow \Delta\theta = \frac{V^2}{4\,JS}$

13 **(b)**

$\tau = MB\sin\theta = 0.1 \times 3 \times 10^{-4}\sin 30°$
$= 1.5 \times 10^{-5}$ N-m

14 **(a)**
The result has to be in one significant umber only.

15 **(c)**
Inside a conducting body, potential is same everywhere and equals to the potential of it's surface

16 **(a)**
If a dielectric slab is inserted between the plates of a charged capacitor, the intensity of electric field potential difference of capacitor and the energy stored all reduce to $\frac{1}{K}$ times and capacity of the capacitor increases $K$ times. But the charge on the capacitor remains unchanged.
Here, $K$ is the dielectric constant of dielectric.

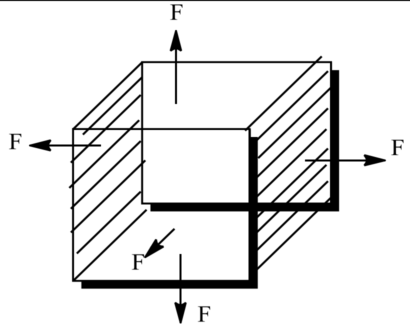
17 **(c)**
At $TK$, pressure of gas $(P)$ in the jar
= Total pressure – saturated vapour pressure
$\Rightarrow P = (830 - 30) = 800 mm\ of\ Hg$
New temperature $T' = \left(T - \frac{T}{100}\right) = \frac{99T}{100}$
Using Charle's law $\frac{P}{T} = \frac{P'}{T'} \Rightarrow P' = \frac{PT'}{T}$
$= \frac{800 \times 99T}{100T} = 792 mm\ of\ Hg$
Saturated vapour pressure at $T' = 25\ mm\ of\ Hg$
$\therefore$ Total pressure in the jar
= Actual pressure of gas + Saturated vapour pressure
$= 792 + 25 = 817\ mm$ of $Hg$

18 **(a)**
$$n \propto \sqrt{\frac{k}{m}}$$

19 **(d)**
$2\,Sl = F$
Or $S = F/2l = (2 \times 10^{-2})/2 \times 0.10 = 0.1$ Nm$^{-1}$

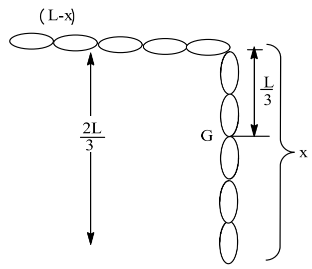
20 **(c)**
When gravitational force becomes zero, then centripetal force on satellite becomes zero and therefore, the satellite will become stationary in its orbit.

21 **(2)**




Tensile strain on each face $= \frac{l}{L} = \frac{FA}{Y}$
$For\ A = 1 sq.unit,$
$\frac{l}{L} = \frac{F}{Y}$
Lateral strain due to force acting on perpendicular face $= -\sigma \times \frac{l}{L}$
$= -\sigma \times \frac{F}{Y}$
As force is subjected from both directions,
$Lateral\ strain = \frac{-2\sigma F}{Y}$
Total increase in length is attributed to both tensile and lateral strain.
$\therefore$ Increase in length of each side
$= \frac{F}{Y} + \frac{-2\sigma F}{Y}$
$= (1 - 2\sigma)\ \frac{F}{Y}$
$\Longrightarrow x = 2$

22 **(19.2)**




The length of hanging part is $\frac{2L}{3}$, so its mass would be $\frac{2M}{3}$ and will act at the centre of gravity G of hanging part, i.e., at a distance $\frac{L}{3}$ below the surface of table.
$\therefore$ Work done in pulling the hanging part on the table = increase in P.E.
$=$ mgh
$= \frac{2M}{3} \times g \times \frac{L}{3}$
$= \frac{2MgL}{3} = \frac{2 \times 0.54 \times 10 \times 16}{3} = 19.2$ J

**Alternate method :**

$\therefore\ W = F\,dx = \frac{M}{L}(X)\,g\,dx$

$\therefore$ Total work done to pull the $\frac{2}{3}^{rd}$ part on the table,

$$W = \int_0^{\frac{2L}{3}} \frac{M}{L} gx\,dx = \frac{M}{L} g \int_0^{\frac{2L}{3}} x\,dx$$

$$= \frac{M}{L} g \left[\frac{x^2}{2}\right]_0^{\frac{L}{3}} = \frac{M}{L} g \left[\frac{\frac{2L}{3}^2}{2}\right]$$

$$= \frac{Mg}{L} \times \frac{4L^2}{18}$$

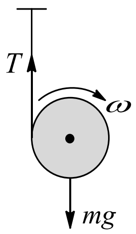
$$\therefore\ W = \frac{2MgL}{9}$$

$$\therefore\ W = \frac{2 \times 0.54 \times 10 \times 16}{9}$$

$\therefore\ W = 19.2$ J

23 **(5)**

Let $T$ be the tension in the thread and $f$, the linear acceleration of the reel as it fails




For the downward translation

$(mg - T) = mf$ (i)

For the rotational motion of the reel, angular acceleration is $\alpha = \left(\frac{f}{a}\right)$ and $T = \frac{mg}{3}$ (ii)

From Eqs (i) and (ii), $T = mg - mf = mg - ma\alpha$

$= mg - 2T$

$\Rightarrow 3T = mg$

$\therefore T = \frac{mg}{3} = 1.5 \times 10/3 = 5$ N

24 **(15)**

Fringe − width, $\beta = \frac{D\lambda}{d}$

Shifting of fringe pattern due to paper sheet,

$S = \frac{t(\mu - 1)D}{d}$

If we paste paper sheet on the other slit, the fringe pattern will shift by same amount on that side. Hence, number of fringes crossing the centre is

$$n = \frac{S}{\beta}$$
$$= \frac{t(\mu - 1)}{\lambda}$$
$$= \frac{0.02 \times 10^{-3} \times (1.45 - 1)}{600 \times 10^{-9}}$$
$$= \frac{0.02 \times 0.45 \times 10^6}{600}$$
$$= 15$$

25 **(6)**

Maximum range $(\theta = 45°) = \frac{u^2}{g} = 0.8\,u = 2\sqrt{2}$ m

Distance covered in 3 s $= (u\cos 45°)(3)$

$= (2\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)(3) = 6$ m

26 **(35)**

By Newton's law of cooling,

$$\frac{\theta_1 - \theta_1}{\Delta t} = K\left(\frac{\theta_1 + \theta_1}{2} - \theta_0\right)$$

For first 10 minutes,

$$\frac{85 - 55}{10} = K(70 - \theta_0)$$

$\therefore 3 = K \times (70 - \theta_0)$ ... (i)

Similarly, for next 10 minutes,

$$\frac{55 - 43}{10} = K(49 - \theta_0)$$

$1.2 = K \times (49 - \theta_0)$ ... (ii)

Dividing equation (i) by equation (ii),

$$\frac{3}{1.2} = \frac{70 - \theta_0}{49 - \theta_0}$$

$2.5(49 - \theta_0) = 70 - \theta_0$

$\therefore 122.5 - 2.5\theta_0 = 70 - \theta_0$

$\therefore 1.5\theta_0 = 52.5$

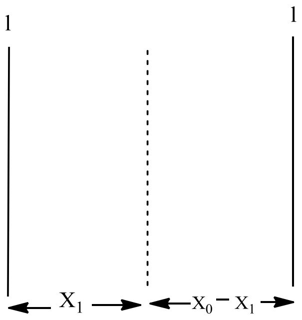
$\therefore \theta_0 = \frac{52.5}{1.5} = 35$ °C

27 **(3)**

$$B_2 = \frac{\mu_0 I}{2\pi x_1} + \frac{\mu_0 I}{2\pi(x_0 - x_1)}$$

(when current are in opposite directions)

$$B_1 = \frac{\mu_0 I}{2\pi x_1} - \frac{\mu_0 I}{2\pi(x_0 - x_1)}$$




(when currents are in same direction)

Subtituting $x_1 = \frac{x_0}{3}$ $\left(\text{as } \frac{x_0}{x_1} = 3\right)$

$$B_1 = \frac{3\mu_0 I}{2\pi x_0} - \frac{3\mu_0 I}{4\pi x_0} = \frac{3\mu_0 I}{4\pi x_0}$$

$$R_1 = \frac{mv}{qB_1}$$

and $B_2 = \frac{9\mu_0 I}{4\pi x_0}$

$$R_1 = \frac{mv}{qB_1}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{9}{3} = 3$$

28 **(2)**

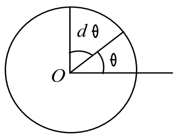
For the equilibrium of charge at $z = a$, the net electric field at this point due to both the ring should be zero

$$E_A + E_B = 0 \Rightarrow \frac{kq_A a}{(a^2 + a^2)^{3/2}} + \frac{kq_B a}{(b^2 + a^2)^{3/2}} = 0$$

Put the values and solve to get $b/a = 2$

29 **(0)**

The charge on the infinitesimal elements of arc which subtend an angle $d\theta$ at the centre of the ring




$$dQ = \lambda R d\theta = \lambda_0 \cos\frac{\theta}{2}\ R d\theta$$

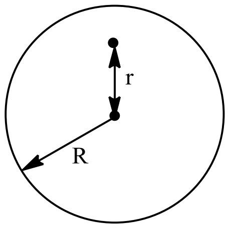
Potential at the centre of ring due to charge $dQ$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R} = \frac{\lambda_0 \cos\frac{\theta}{2}.Rd\theta}{4\pi\varepsilon_0 R}$$

$$V = \int dV \Rightarrow V = \frac{\lambda_0}{4\pi\varepsilon_0}\int_0^{2\pi} \cos\frac{\theta}{2} d\theta$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0}\left[\frac{\sin\frac{\theta}{2}}{\frac{1}{2}}\right]_0^{2\pi} = 0 \text{ V}$$

30 **(0.3)**




$$T = 2\pi\sqrt{\frac{1}{mgh}}$$

$$I = \frac{mR^2}{2} + mr^2$$

$$= \frac{mR^2}{2} + m\left(\frac{R}{4}\right)^2$$

$$= \frac{mR^2}{2} + \frac{mR^2}{16}$$

$$= \frac{9}{16}mR^2$$

Here, $R = 10 \text{ cm} = 0.1 \text{ m}, h = \frac{R}{4}$

$$\Rightarrow T = 2\pi\sqrt{\frac{\frac{9}{16}mR^2}{\frac{mgR}{2}}} = 2\pi\sqrt{\frac{9R}{4g}} = 2\pi\sqrt{\frac{9 \times 0.1}{4 \times 10}}$$

$$\therefore T = 2\pi \times \frac{3}{2} \times \frac{1}{10} = 0.3\pi s$$

## : ANSWER KEY :

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 61) | a | 62) | b | 63) | c | 64) | c | 81) | 100 | 82) | 4 | 83) | 3 | 84) | 4 |
| 65) | d | 66) | a | 67) | c | 68) | c | 85) | 0 | 86) | 4 | 87) | 3 | 88) | 8 |
| 69) | b | 70) | c | 71) | c | 72) | a | 89) | 1 | 90) | 1 | | | | |
| 73) | c | 74) | c | 75) | a | 76) | a | | | | | | | | |
| 77) | c | 78) | d | 79) | b | 80) | c | | | | | | | | |

# : HINTS AND SOLUTIONS :

61 **(a)**

The number formed will be divisible by 4 if the number formed by the two digits on the extreme right is divisible by 4 i.e. it should be 12,24,32,52,44

The number of numbers ending in 12 $= 5 \times 5$

The number of numbers ending in 24 $= 5 \times 5$

The number of numbers ending in 32 $= 5 \times 5$

The number of numbers ending in 52 $= 5 \times 5$

The number of numbers ending in 44 $= 5 \times 5$

Thus, the required number

$= 5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5 = 125$

62 **(b)**

We have,

$\alpha_1\alpha_2 = \beta_1\beta_2 = 1 \Rightarrow \alpha_1 = \frac{1}{\alpha_2}$ and $\beta_1 = \frac{1}{\beta_2}$

This means that the roots of the equation $a_2x^2 + b_2x + c_2 = 0$ are reciprocal of the roots of the equation $a_1x^2 + b_1x + c_1 = 0$

Therefore, equations $a_1x^2 + b_1x + c_1 = 0$ and $c_2x^2 + b_2x + a_2 = 0$ have same roots

$\therefore \frac{a_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{a_2}$

63 **(c)**

$\alpha$ and $\beta$ are roots of the equation

$x^2 - x + 1 = 0$

$\Rightarrow \alpha + \beta = 1, \alpha\beta = 1$

$\Rightarrow \alpha = -\omega, \beta = -\omega^2$

or $\alpha = -\omega^2, \beta = -\omega$

Taking $\alpha = -\omega, \beta = -\omega^2$

$\alpha^{2009} + \beta^{2009} = (-\omega)^{2009} - (-\omega^2)^{2009}$

$= -(\omega^2 + \omega)$

$= 1$

64 **(c)**

The given plane passes through $\vec{a}$ and is parallel to the vectors $(\vec{b} - \vec{a})$ and $\vec{c}$. So it is normal to $(\vec{b} - \vec{a}), \times \vec{c}$

$(\vec{r} - \vec{a}) \cdot ((\vec{b} - \vec{a}) \times \vec{c}) = 0$

$\Rightarrow \vec{r} \cdot (\vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = [\vec{a}\,\vec{b}\,\vec{c}]$

The length of the perpendicular from the origin to this plane is

$$\frac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$$

65 **(d)**

Since, $P + \lambda P' = 0$ ....(i)

$\Rightarrow ax + by + cz + d + \lambda(a'x + b'y + c'z + d') = 0$

For parallel to $x - axis$, coefficient of $x = 0$

$\Rightarrow a + \lambda a' = 0 \Rightarrow \lambda = -\frac{a}{a'}$

$\therefore$ From Eq. (i), we get

$P - \frac{a}{a'}P' = 0$

$\Rightarrow \frac{P}{a} = \frac{P'}{a'}$

66 **(a)**

Let the locus of point be $(x, y)$

Area of triangle with points $(x, y)$, $(1, 5)$ and $(3, -7)$ is 21 sq unit

$\therefore \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 5 & 1 \\ 3 & -7 & 1 \end{vmatrix} = 21$

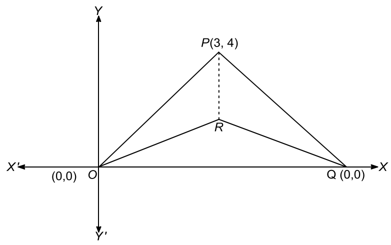
$\Rightarrow \frac{1}{2}[x(5 + 7) - y(1 - 3) + 1(-7 - 15] = 21$

$\Rightarrow \frac{1}{2}[12x + 2y - 22] = 21$

$\Rightarrow 6x + y - 32 = 0$

67 **(c)**

Since, triangle is isosceles, hence centroid is the desired point




$\therefore$ Coordinates of $R\left(3, \frac{4}{3}\right)$.

68 **(c)**

$\cos(\theta + \phi) = m\cos(\theta - \phi)$

$\Rightarrow \cos\theta\cos\phi - \sin\theta\sin\phi = m\cos\theta\cos\phi + m\sin\theta\sin\phi$

$\Rightarrow \cos\theta\cos\phi\,(1 - m) = \sin\theta\sin\phi(1 + m)$

$\Rightarrow \tan\theta = \left[\frac{1 - m}{1 + m}\right]\cot\phi$

69 **(b)**

Given, $y = a\sin^3\theta$ and $x = a\cos^3\theta$

On differentiating w.r.t. θ, we get

$\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$

and $\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta} = -\tan\theta$

At $\theta = \frac{\pi}{3}, \frac{dy}{dx} = -\tan\frac{\pi}{3} = -\sqrt{3}$

70 **(c)**

Let $P(x, y)$ be the point on the curve $x^2 = 2y$ such that it is nearest to the point $A(0, 3)$. Then,

$AP^2 = x^2 + (y-3)^2$

$\Rightarrow AP^2 = 2y + (y-3)^2 = y^2 - 4y + 9$

$= (y-2)^2 + 5 \geq 5$

Clearly, $AP^2$ is minimum when $y = 2$ and the minimum value of $AP$ is $\sqrt{5}$

Putting $y = 2$ in $x^2 = 2y$ we get $x = \pm 2$

Hence, the required points are $(2, 2)$ and $(-2, 2)$

71 **(c)**

Given, $S_n = 1^3 + 2^3 + \ldots + n^3 = \Sigma n^3$

and $T_n = 1 + 2 + \ldots + n = \Sigma n$

$\because S_n = \Sigma n^3 = \left[\frac{n(n+1)}{2}\right] \Rightarrow S_n = \{\Sigma n\}^2 = T_n^2$

72 **(a)**

$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$\Rightarrow \frac{3x + 2x}{1 - 6x^2} = \frac{\pi}{4}$

$\Rightarrow 5x = 1 - 6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow x = -1, \frac{1}{6}$

But when $x = -1$,

$\tan^{-1} 2x = \tan^{-1}(-2) < 0$

And $\tan^{-1} 3x = \tan^{-1}(-3) < 0$

This value will not satisfy the given equation

Hence, $x = \frac{1}{6}$

73 **(c)**

We have, $|2x - 3| < |x + 5|$

$\Rightarrow |2x - 3| - |x + 5| < 0$

$$\Rightarrow \begin{cases} 3 - 2x + x + 5 < 0, x \leq -5 \\ 3 - 2x - x - 5 < 0, -5 < x \leq \frac{3}{2} \\ 2x - 3 - x - 5 < 0, x > \frac{3}{2} \end{cases}$$

$$\Rightarrow \begin{cases} x > 8, x \leq -5 \\ x > -\frac{2}{3}, -5 < x \leq \frac{3}{2} \\ x < 8, x > \frac{3}{2} \end{cases}$$

$\Rightarrow x \in \left(-\frac{2}{3}, \frac{3}{2}\right] \cup \left(\frac{3}{2}, 8\right)$

$\Rightarrow x \in \left(-\frac{2}{3}, 8\right)$

74 **(c)**

For $f(x)$ to be continuous at $x = 0$, we must have

$\lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \lim_{x \to 0} \frac{(9^x - 1)(4^x - 1)}{\sqrt{2} - \sqrt{2\cos^2 x/2}} = k$

$\Rightarrow \lim_{x \to 0} \frac{(9^x - 1)(4^x - 1)}{\sqrt{2}.\, 2\sin^2 x/4} = k$

$\Rightarrow \lim_{x \to 0} \frac{16 \times \left(\frac{9^x - 1}{x}\right)\left(\frac{4^x - 1}{x}\right)}{2\sqrt{2}\left(\frac{\sin x/2}{x/4}\right)^2} = k$

$\Rightarrow \frac{16}{2\sqrt{2}} \log 9 \,.\, \log 4 = k = 4\sqrt{2} \log 9 \,.\, \log 4$

$= 16\sqrt{2} \log 3 \log 2$

75 **(a)**

The general term of the given series is

$T_r = (-1)^r (3 + 5r)\, {}^nC_r$

$\therefore$ Sum $\sum_{r=0}^{n} (1)^r\, (3 + 5r)\, {}^nC_r$

$= 3 \;{}^{\;n}_{r=0}\; (-1)^r\, {}^nC_r + 5\sum_{r=0}^{n} (1)^r\; r\, {}^nC_r$

$= 3(C_0 - C_1 + C_2 - C_3 + C_4 - \ldots + (-1)^n \cdot C_n)$

$+5(-C_1 + 2C_2 - 3C_3 + 4C_4 - \cdots + (-1)^n \cdot n \cdot C_n)$

$\Longrightarrow S = 0 + 0 = 0$

76 **(a)**

Suppose $x^7$ occurs in $(r+1)^{th}$ term in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$

We have,

$T_{r+1} = {}^{11}C_r (a\, x^2)^{11-r} \left(\frac{1}{b\, x}\right)^r$

$= {}^{11}C_r\, a^{11-r}\, b^{-r} x^{22-3r}$

This will contain $x^7$, if

$22 - 3r = 7 \Rightarrow r = 5$

$\therefore$ Coefficient of $x^7$ in $(ax^2 + b^{-1}\, x^{-1})^{11}$

$= {}^{11}C_5 a^6 b^{-5}$

Let $x^{-7}$ occur in $(s+1)^{th}$ term of the expansion of $\left(ax - \frac{1}{b\, x^2}\right)^{11}$

We have,

$T_{s+1} = {}^{11}C_s\, (a\, x)^{11-s} \left(-\frac{1}{b\, x^2}\right)^s$

$\Rightarrow T_{s+1} = {}^{11}C_s\, a^{11-s} b^{-s} (-1)^s\, x^{11-3s}$

This is contain $x^{-7}$, if

$11 - 3s = -7 \Rightarrow s = 6$

$\therefore$ Coefficient of $x^{-7}$ in $(ax - b^{-1}\, x^{-2})^{11}$

$= {}^{11}C_6\, a^5 b^{-6}$

It is given that

${}^{11}C_6\, a^5 b^{-6} = {}^{11}C_5 a^6 b^{-5} \Rightarrow ab = 1$

77 **(c)**

$(\lambda AB + 3BA)^T = \lambda B^T A^T + 3A^T B^T$

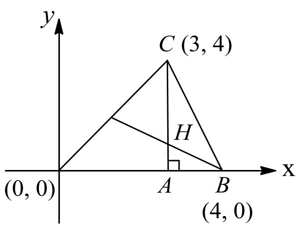
$(\lambda AB + 3BA)^T = -\lambda BA - 3AB$

$(\lambda AB + 3BA)^T = -(3AB + \lambda BA)$

$\lambda = 3$

78 **(d)**

Let $H(3, \alpha)$ is the orthocenter




$\therefore$ Slope of $BH \times$ Slope of $AC = -1$

$\Rightarrow -\alpha.\dfrac{4}{3} = -1$

$\Rightarrow \alpha = \dfrac{3}{4}$

Hence, orthocenter of a triangle is $\left(3, \frac{3}{4}\right)$

79 **(b)**

Let $I = \int_5^{10} \frac{1}{(x-1)(x-2)} dx$

$$\int_5^{10} \left[\frac{-1}{x-1} + \frac{1}{x-2}\right] dx$$

$= [-\log(x-1) + \log(x-2)]_5^{10}$

$= -\log 9 + \log 8 + \log 4 - \log 3$

$= -2\log 3 + 3\log 2 + 2\log 2 - \log 3$

$= -3\log 3 + 5\log 2$

$= -\log 27 + \log 32$

$= \log \dfrac{32}{27}$

80 **(c)**

Let $I = \int_0^{\pi/2} \log \sin x \, dx$ ... (i)

$$I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$= \int_0^{\pi/2} \log \cos x \, dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} (\log \sin x + \log \cos x) dx$$

$$= \int_0^{\pi/2} \log \sin 2x + dx - \log 2 \int_0^{\pi/2} dx$$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

$$\therefore 2I = \int_0^{\pi} \frac{\log \sin t}{2} dt - \log 2 [x]_0^{\pi/2}$$

$$= \frac{1}{2} \times 2 \int_0^{\pi/2} \log \sin t \, dt - \frac{\pi}{2} \log 2$$

$$= \int_0^{\pi/2} \log \sin x dx - \frac{\pi}{2} \log 2$$

$$= I - \frac{\pi}{2} \log 2$$

$$\Rightarrow I = -\frac{\pi}{2} \log 2$$

81 **(100)**

Total number of numbers of not more than 20 digits

$= 5 + 4(5) + 4(5)^2 + 4(5)^3 + \cdots + 4(5)^{19}$

$= 5 + 4 \times 5 \left(\dfrac{5^{19} - 1}{5 - 1}\right)$

$= 5 + 5^{20} - 5 = 5^{20}$

$\Rightarrow m = 5, n = 20 \Rightarrow mn = 100$

82 **(4)**

$$5 \frac{2 \tan \beta}{1 + \tan^2 \beta} = 3 \frac{2 \tan \alpha}{1 + \tan^2 \alpha}$$

$$\Rightarrow \frac{5 \tan \beta}{1 + \tan^2 \beta} = \frac{3 \tan \alpha}{1 + \tan^2 \alpha}$$

Substituting $\tan \beta = 3 \tan \alpha$, we have

$$\frac{5 \times 3 \tan \alpha}{1 + 9 \tan^2 \alpha} = \frac{3 \tan \alpha}{1 + \tan^2 \alpha}$$

$\Rightarrow 5 + 5 \tan^2 \alpha = 1 + 9 \tan^2 \alpha$

$\Rightarrow 4 \tan^2 \alpha = 4$

$\Rightarrow \tan \alpha = 1, \text{i.e.}, \tan \beta = 3$

$\therefore \tan \alpha + \tan \beta = 4$

83 **(3)**

$$y = \frac{x^4 - (x^2 + 2x + 1)}{x^2 - x - 1} = x^2 + x + 1$$

$\therefore \dfrac{dy}{dx} = 2x + 1 = ax + b$

Hence $a = 2$ and $b = 1$

84 **(4)**

$f(x) = \left(\frac{1}{x}\right)^x = x^{-x}$

Differentiating with respect to x,

$f'(x) = -x^{-x}(1 + \log x)$

.. [By logarithmic differentiation]

$f'(x) = 0$

$\Leftrightarrow 1 + \log x = 0$

$\Leftrightarrow x = \dfrac{1}{e}$

$f'$ ——— ( + • − ) ———

$x = \frac{1}{e}$ (a point of relative maxima)

Maximum value of the function $= (e)^{\frac{1}{e}}$

$\Rightarrow p = e, q = \dfrac{1}{e}$

$\Rightarrow [p] + \left[\dfrac{1}{q}\right] = [e] + [e] = 2 + 2 = 4$

85 **(0)**

$\sin A + \cos A = m$

$\Longrightarrow (\sin A + \cos A)^3 = m^3$

$\Leftrightarrow \sin^3 A + \cos^3 A + 3 \sin A \cos A (\sin A + \cos A)$
$= m^3$

$\Leftrightarrow n + 3 \sin A \cos A\,(m) = m^3 \quad \ldots (i)$

Also, $m^2 = (\sin A + \cos A)^2$
$= \sin^2 A + \cos^2 A$
$= 1 + 2 \sin A \cos A$

$\Longrightarrow \sin A \cos A = \dfrac{m^2 - 1}{2}$

Substituting $\sin A \cos A$ in (i), we get

$n + 3m\left(\dfrac{m^2 - 1}{2}\right) = m^3$

$\Longrightarrow m^3 - 3m + 2n = 0$

86 **(4)**

$$\Delta = x\begin{vmatrix} 1 & x+y & x+y+z \\ 2 & 3x+2y & 4x+3y+2z \\ 3 & 6x+3y & 10x+6y+3z \end{vmatrix}$$

$$= x^2\begin{vmatrix} 1 & 1 & x+y \\ 2 & 3 & 4x+3y \\ 3 & 6 & 10x+6y \end{vmatrix} \begin{vmatrix} C_3 \to C_3 - zC_1 \\ C_2 \to C_2 - yC_1 \end{vmatrix}$$

$$= x^3\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix} [C_3 \to C_3 - yC_2]$$

$= x^3(6 - 8 + 3) = 64$

$= x^3(6 - 8 + 3) = 64$

$\Rightarrow x^3 = 64 \Rightarrow x = 4$

87 **(3)**

$369 = \dfrac{9}{2}[2 + (9 - 1)d]$

$\Rightarrow 82 = 2 + 8d$

$\Rightarrow d = 10$

Now $ar^8 = a + 8d = 1 + 8 \times 80 = 81$

$\Rightarrow r^8 = 81$

$\Rightarrow r = \sqrt{3}$

$\Rightarrow ar^{(7-1)} = 1 \times \left(\sqrt{3}\right)^6 = 27$

88 **(8)**

$\log_2 x + \log_2 \sqrt{x} + \log_2 \sqrt[4]{x} + \cdots = 4$

$\Rightarrow \log_2 \left(x^{1+\frac{1}{2}+\frac{1}{4}\pm}\right) = 4$

$\Rightarrow \log_2 \left(x^{1-\frac{1}{2}}\right) = 4$

$\Rightarrow \log_2 x^2 = 4 \Rightarrow x^2 = 2^4$

$\Rightarrow x^2 = 4^2$

$\Rightarrow x = 4$ ....[x>0]

$x + 4 = 4 + 4 = 8$

89 **(1)**

Since angle between $\vec{u}$ and $\hat{\imath}$ is 60°, we have

$\vec{u} \cdot i = |\vec{u}||\hat{\imath}| \cos 60° = \dfrac{|\vec{u}|}{2}$

Given that $|\vec{u} - \hat{\imath}|, |\vec{u}|, |\vec{u} - 2\hat{\imath}|$ are in G.P., so

$|\vec{u} - \hat{\imath}|^2 = |\vec{u}||\vec{u} - 2\hat{\imath}|$

Squaring both sides,

$[|\vec{u}|^2 + |\hat{\imath}|^2 - 2\vec{u} \cdot \hat{\imath}]^2 = |\vec{u}|^2[|\vec{u}|^2 + 4|\hat{\imath}|^2 - 4\vec{u} \cdot \hat{\imath}]$

$\left[|\vec{u}|^2 + 1 - \dfrac{2|\vec{u}|}{2}\right]^2 = |\vec{u}|^2\left[|\vec{u}|^2 + 4 - 4\dfrac{|\vec{u}|}{2}\right]$

Or $|\vec{u}|^2 + 2|\vec{u}| - 1 = 0 \Rightarrow |\vec{u}| = -\frac{2\pm2\sqrt{2}}{2}$

Or $|\vec{u}| = \sqrt{2} - 1$

90 **(1)**

Let $y(x) = [f(x)]^3 + [g(x)]^3 + [h(x)]^3$
$- 3f(x)g(x)h(x)$

$\Longrightarrow y'(x) = 3\,[f(x)]^2 f'(x) + 3[g(x)]^2 g'(x)$
$+ 3[h(x)]^2 h'(x)$
$- 3\big[f(x)g(x)h'^{(x)} + f(x)g'(x)h(x)$
$+ f'(x)g(x)h(x)\big]$

$= 3[f(x)]^2 g(x) + 3[g(x)]^2 h(x) + 3[h(x)]^2 f(x)$
$- 3[f(x)g(x)f(x) + f(x)h(x)h(x)$
$+ g(x)g(x)h(x)]$

$= 0$

$\Longrightarrow y(x)$ is a constant for all $x \in R$

$\Longrightarrow y(7) = y(0)$

$y(0) = [f(x)]^3 + [g(x)]^3 + [h(x)]^3$
$- 3f(0)g(0)h(0)$

$= 1^3 + 0 + 0 - 3(0)$

$= 1$

$\Longrightarrow y(7) = 1$